PHYSICAL REVIEW E **78**, 010302(R) (2008)

Gaussian distributions of rotational velocities in a granular medium

Malte Schmick and Mario Markus*
Max-Planck-Institut für Molekulare Physiologie, Postfach 500247, 44202 Dortmund, Germany
(Received 23 March 2008; published 23 July 2008)

We investigate experimentally a homogeneously driven granular medium. It consists of spheres, each one containing a magnetic dipole, rolling on a horizontally placed dish and subject to a magnetic field which is sinusoidal in time and spatially homogeneous. A gaslike state is obtained. Except for extremely low amplitudes of the magnetic field, non-Gaussian distributions are found for the translatory velocities, as well as for the horizontal components of the rotational velocities. However, the vertical component of the rotational velocity obeys an almost invariant Gaussian distribution within a large range of parameters, namely frequencies and amplitudes of the magnetic field and friction coefficients on the dish's bottom. The results are close to predictions from simulations by Cafiero *et al.* [Europhys. Lett. **60**, 854 (2002)].

DOI: 10.1103/PhysRevE.78.010302 PACS number(s): 45.70.−n, 51.10.+y, 07.05.Hd

I. INTRODUCTION

Velocity distributions in granular media are, in general, non-Gaussian. One reason is spatial clustering caused by dissipative collisions [1]; another reason is the supply of energy in a spatially inhomogeneous way [2]. However, by partly obviating these features of granular systems, some observations of Gaussian distributions have been reported. A first experiment was made by Ippolito *et al.* [3,4], placing disks on a porous plane through which air flowed from below. Later on, Olafsen and Urbach [5] obtained a Gaussian distribution for spheres on a vibrating plate, provided the input energy is sufficiently large so that the effect of inelastic collisions is negligible. Another experiment was reported by Prevost *et al.* [6] for a sufficiently low density of spheres on a vibrating plate that is so rough that a distinguished direction of driving is prevented. Still another observation is due to Baxter and Olafsen [7], who placed a bed of heavier grains between the vibrating plate and the investigated medium, these heavier grains acting as an extremely uneven "boundary."

The present work was inspired by the simulations of Cafiero *et al.* [8]; they showed that a rotationally driven granular medium displays a Gaussian distribution of the rotational velocities, while a non-Gaussian one is obtained for the translational velocities. The rotational driving is performed in our setup by an alternating magnetic field interacting with magnetic dipoles embedded in spheres. Sliding friction of the spheres with the supporting horizontal plane transduces rotational into translational energy, while collisions between spheres randomize the system. These collisions also induce the rotation around the vertical axis; the magnetic field solely drives rotations in the x-y plane.

Hitherto, a number of works have reported on the formation of chains, rings, and other patterns in systems of magnetic microspheres (see, e.g., [9,10]). For larger particles there are reports on the influence of a static magnetic field on the angle of repose of magnetizable particles in a rotating drum [11] or on clustering transitions in vibrated containers [12]. In addition, pattern formation of shaken magnetic spheres [13,14], as well as of floating magnetic dipoles above a rotating permanent magnet [15] were reported. In contrast to these works, we shall not concentrate here on pattern formation, but on randomized states.

Since in our setup the magnetic field is spatially constant and interacts with each sphere, we transfer energy individually and homogeneously to the medium. Thus, our system is to be described as "uniformly heated," as opposed to "heated by the boundary" [2]. This condition makes our system a promising candidate for a Gaussian distribution [1]. Moreover, our work yields rotational velocities and their distribution measured in a granular medium.

II. EXPERIMENTAL METHODS

Figure 1 shows a rough sketch of the illumination (upper part) and the dish with the magnetic spheres (lower part). We used a Petri dish (diameter, 5 cm; height, 7 mm). If not stated otherwise, the bottom of the dish was covered with black paper. We used spheres with encased magnets, comparable to the ones used in the works by Stambaugh *et al.* [13,14]. In our case, the medium consisted of 10 white wood spheres (radius, R_S=3 mm), each of which was constructed by gluing together two hemispheres with holes drilled so as to hold a well-centered magnet. Magnets were obtained by punching round holes (with a ticket punch; diameter, 2 mm) into magnetic rubber plates (Flexo 180 from *Schallenkammer*, Rimpar, Germany; thickness, 1.5 mm; remanence, 265 mT; length of magnet, 2×1.5 mm). The dish was illu-

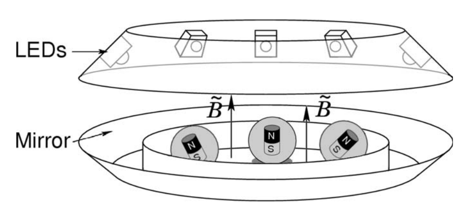

FIG. 1. Rough sketch of the experimental setup. $\tilde{B}$, sinusodial magnetic field. For clarity, only three spheres with embedded dipoles are shown and their size is exaggerated.

*markus@mpi-dortmund.mpg.de

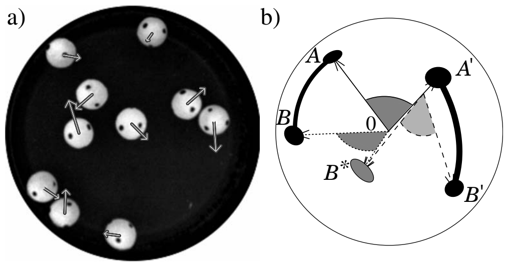

FIG. 2. (a) Typical snapshot of the spheres in the dish. The arrows indicate the translational velocity. (b) Enlarged scheme of one sphere explaining the determination of its rotation. In subsequently recorded pictures, A moves to A' and B moves to B' (note, these transitions are shown exaggeratedly large here). The net rotation is composed of the shortest path from A to A', transforming B into B^* (dark shaded angle), followed by the rotation from B^* to B' around the axis $\overline{0A'}$ (light shaded angle).

minated from above by three conical layers of 40 LEDs each. The light intensity was enhanced by placing a mirror (aluminum foil) around the dish (see Fig. 1). The center of the dish's bottom was placed at the center of a coil (with a vertically aligned axis) generating a sinusodial magnetic field. The coil had 20 cylindrical, concentric layers of copper wire (number of circular windings per layer, 40; wire diameter, 0.27 cm, including 0.2 mm of isolating material; thus, height of the coil, 40×2.7 mm$=10.8$ cm). The inner radius of the coil was 6.75 cm and the outer one was 12.25 cm. An alternating current lead to a magnetic field $\tilde{B}=B_0 \sin(2\pi\nu t)$. Measurements with a Hall probe, which were corroborated by calculations using the Biot-Savart equation, showed that the vertical field component B_z varied less than 2% in radial direction within the Petri dish and less than 0.4% in vertical direction within the diameter of the sphere. In addition, calculations yielded $B_r/B_z < 1.2\%$. The measured inductance of the coil was 88.1 ± 0.1 mH.

The movement of the spheres was monitored by a high speed camera connected to a PC via Firewire. The time difference between succesive images was 4 ms; 3×10^5 images were evaluated for each probability distribution function. A typical image is shown in Fig. 2(a). Translational velocities were determined by tracking the centers of the white spheres on the black background. For the determination of the angular velocities $\vec{\omega}$ we marked six black dots on each sphere: Two at the poles and four, equidistantly, at the equator. The method for the determination of $\vec{\omega}$ is illustrated in Fig. 2(b) where A and B are dots on an image, which move to A' and B' in the following image. Note that it is not sufficient to evaluate just the movement from A to A' because an infinite number of rotations can cause this movement. Thus, the rotation with the smallest angle moving A to A' (which moves B to B^*) was performed first, followed by the rotation around the axis passing through the sphere's center and A' that transforms B^* to B'; the concatenation of those rotations results in the net rotation which transforms the line $\overline{AB}$ to $\overline{A'B'}$.

III. RESULTS AND DISCUSSION

If B_0 is sufficiently low, we observed clustering into chains and rings. If B_0 and ν are sufficiently large the spheres tend to rotate on the spot with the frequency of the applied field, i.e., hardly any transduction of rotation into translation takes place. For intermediate values of B_0 and ν, the torque on the encased magnets causes rolling (without sliding) of the spheres on the ground, while no clustering occurs; this situation applies to parameters chosen in the present work, namely 1.5 mT$\leqslant B_0 \leqslant$4.5 mT and 8 Hz$\leqslant \nu \leqslant$12 Hz.

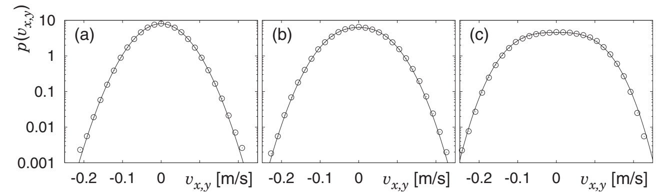

FIG. 3. Probabilities of the translational velocity components v_x and v_y. ν=10 Hz. Curves, fits using Eq. (1). Field amplitude B_0, flatness F and parameter n, (a) B_0=1.5 mT, F=3.05, n=2.000$\pm$0.004; (b) B_0=3 mT, F=2.77, n=2.31$\pm$0.02; (c) B_0=4.5 mT, F=2.39, n=3.00$\pm$0.12.

For all measurements of probability distributions we determined the spatial residence distribution of the spheres in the dish. In doing so, we found an enhanced residence probability near the border of the dish, which is explained by dissipative collisions with the wall, causing a slowing down in its neighborhood. However, by excluding a ring along the edge with a width equal to $1.5R_S$, no deviations from homogeneity of residence were observed; all evaluations in this work were done in the remaining inner area. We also did experiments with square instead of round dishes; in such experiments the spheres accumulated at the corners, thus causing a self-organization of the system into a circularly bordered shape. Therefore, we only report on experiments with the round dish here. Figure 3 (respectively, Fig. 4) shows distributions for the horizontal components of the translational (respectively, rotational) velocities. Figure 5 shows distributions of the vertical component of the rotational velocity. All the measurements in Figs. 3–5 represent probability densities, i.e., the integral probability equals 1.

The collision frequencies vary between 28 Hz (B_0=1.5 mT, ν=10 Hz) and 47 Hz (B_0=4.5 mT, ν=10 Hz), averaged over each time series. In order to check that the initial conditions had no influence on the results, we determined all velocity distributions (Figs. 3–5) for the first one-half of the total measuring time, as well as for the second one-half; we obtained curves having (aside from reduced statistical accuracy) the same shapes. For a quantification of the distributions we fitted the data to the equation

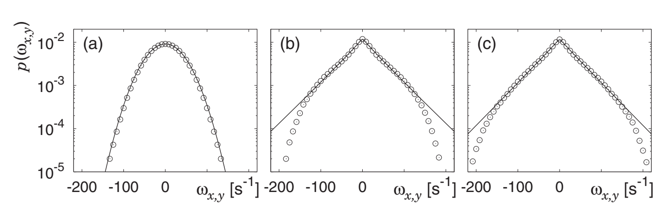

FIG. 4. Probabilites of the rotational velocities ω_x and ω_y. ν=10 Hz. Curves, fits using Eq. (1), (a) B_0=1.5 mT, F=3.07, n=2.08$\pm$0.07; (b) B_0=3 mT, F=5.1, n=1.03$\pm$0.07; (c) B_0=4.5 mT, F=5.9, n=0.98$\pm$0.02; (b,c) fits and kurtosis F are given here for $-100 \leqslant \omega \leqslant 100$ s^{-1}.

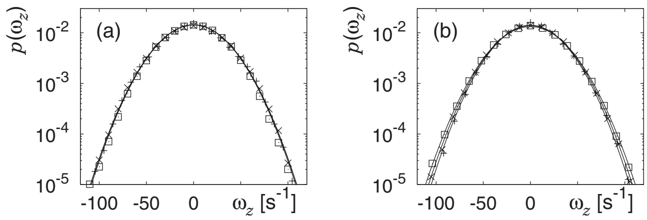

FIG. 5. Almost invariant probabilities of the vertical component ω_z of the rotational velocity. Curves, fits using Eq. (1), (a) $\nu=10$ Hz; +, $B_0=1.5$ mT; ×, $B_0=3$ mT; □, $B_0=4.5$ mT, (b) $B_0=2$ mT; +, $\nu=8$ Hz; ×, $\nu=10$ Hz; □, $\nu=12$ Hz.

$$P(x) = a\exp(-|kx^n|). \tag{1}$$

We expect $n=2$ for a Gaussian distribution and $n=1$ for a Laplacian distribution [16]. Also, we determined the kurtosis (flatness)

$$F = \frac{\langle v\rangle^4}{\langle v^2\rangle^2}. \tag{2}$$

$F=3$ is expected for a Gaussian and $F=6$ for a Laplacian distribution. The values of n and F, which we determined in Figs. 3 and 4, are given in the captions of these figures. The value $n=1.5$, which is often cited as typical for granular media (see, e.g., [17,18]) is not typical in the present work. In contrast, we obtain other values, including $n\approx 1$ (Laplacian distributions), e.g., in Figs. 4(b) and 4(c) for small rotational velocities ($-100 \leqslant \omega_{x,y} \leqslant 100$ s^{-1}).

We obtain Gaussian distributions of $v_{x,y}$ and $\omega_{x,y}$ only for extremely low values of the field amplitude [Figs. 3(a) and 4(a)]. For ω_z, in contrast, Gaussian distributions are obtained for large intervals of the field's amplitude and frequency (Fig. 5). The similarity among the six plots in Fig. 5, as well as their parabolic shape, is astonishing; the distribution is indeed almost invariant to the control parameters. In order to check how robust this finding is with respect to the friction coefficient at the dish's bottom, we replaced the paper by billiard felt and by Parafilm and determined the distribution of ω_z for both differing coverings and the six cases in Fig. 5. Under all these 6×3 conditions, n did not differ from 2 by more than 1%, F did not differ from 3 by more than 3%, and k did not differ from 6.3×10^{-4} by more than 5%, thus showing that the Gaussian distribution is independent of the field's parameters and of friction, as far as variations were performed. In particular, it is remarkable that the granular temperature corresponding to ω_z, which is proportional to $1/k$, is nearly independent of the varied control conditions.

Our results correspond to the work by Cafiero *et al.* [8], who predicted a non-Gaussian distribution for $v_{x,y}$, but a Gaussian for a scalar rotational velocity under stochastic rotational excitation. Note, however, that our experiments are slightly different from these calculations, insofar as in our case the excitation was periodical in the rotational x,y direction and random oblique collisions between spheres induced the rotational velocity ω_z. It is remarkable that in comparison with previous reports of Gaussian distributions in granular media [3–7], we obtained such distributions in surprisingly large intervals of control parameters.

A result requiring an explanation is the appearance of Gaussian distributions at low driving energies (low B_0) in Figs. 3 and 4, while for vibrated media such distributions appear for large input energies [5,19]. In this context, we must clarify that below our lowest B_0 ($B_0 \lesssim 1$ mT) we obtain clustering into chains and rings and thus no Gaussian distributions. On the other hand, if we increase B_0 above 5 mT the torque imparted to the dipoles by the field becomes much larger than the angular momentum exchanged by the spheres in collisions; this causes an enhanced synchronization of the dipole dynamics with the field and a reduced randomization by collisions. For intermediate values of B_0 ($1.5\text{ mT} \leqslant B_0 \leqslant 4.5$ mT) we observe only rolling of the spheres, such that the resulting horizontal movement causes sufficient collisions (more than 33 000 for each distribution function) to randomize the system. This yielded Gaussian distributions for v_x, v_y, ω_x, and ω_y at $B_0=1.5$ mT, as well as for ω_z over the whole range of B_0. In contrast to this, vertically vibrated media (see experiments in [5] and calculations in [19]) yield non-Gaussian distributions at lower excitation strengths because the vertical motion dominates over the horizontal one (then, the particles nearly follow the movement of the plate), while for sufficiently high excitation strengths, collisions randomize the horizontal movement of the particles, so as to render a Gaussian distribution.

ACKNOWLEDGMENTS

We thank Oliver Schulz for his valuable help in the algorithmic evaluation of data. Also, we thank the devoted assistance of Stefan Beckmann in his precise construction of the granular medium. This work was financially supported by the Deutsche Forschungsgemeinschaft (Grant No. MA 629/6-).

[1] A. Puglisi, V. Loreto, U. Marini Bettolo Marconi, and A. Vulpiani, Phys. Rev. E **59**, 5582 (1999).
[2] J. S. van Zon and F. C. MacKintosh, Phys. Rev. E **72**, 051301 (2005).
[3] I. Ippolito, C. Annic, J. Lemaitre, L. Oger, and D. Bideau, Phys. Rev. E **52**, 2072 (1995).
[4] L. Oger, C. Annic, D. Bideau, R. Dai, and S. B. Savage, J. Stat. Phys. **82**, 1047 (1996).
[5] J. S. Olafsen and J. S. Urbach, Phys. Rev. E **60**, R2468 (1999).
[6] A. Prevost, D. A. Egolf, and J. S. Urbach, Phys. Rev. Lett. **89**, 084301 (2002).
[7] G. W. Baxter and J. S. Olafsen, Nature (London) **425**, 680 (2003).
[8] R. Cafiero, S. Luding, and H. J. Herrmann, Europhys. Lett. **60**, 854 (2002).
[9] W. Wen, F. Kun, K. F. Pal, D. W. Zheng, and K. N. Tu, Phys.

Rev. E **59**, R4758 (1999).
[10] P. G. deGennes and P. A. Pincus, Phys. Kondens. Mater. **11**, 189 (1970).
[11] A. J. Forsyth, S. R. Hutton, M. J. Rhodes, and C. F. Osborne, Phys. Rev. E **63**, 031302 (2001).
[12] D. L. Blair and A. Kudrolli, Phys. Rev. E **67**, 021302 (2003).
[13] J. Stambaugh, D. P. Lathrop, E. Ott, and W. Losert, Phys. Rev. E **68**, 026207 (2003).
[14] J. Stambaugh, Z. Smith, E. Ott, and W. Losert, Phys. Rev. E **70**, 031304 (2004).
[15] B. A. Grzybowski, H. A. Stone, and G. M. Whitesides, Nature (London) **405**, 1033 (2000).
[16] O. Schulz and M. Markus, J. Phys. Chem. B **111**, 8175 (2007).
[17] F. Rouyer and N. Menon, Phys. Rev. Lett. **85**, 3676 (2000).
[18] T. Antal, M. Droz, and A. Lipowski, Phys. Rev. E **66**, 062301 (2002).
[19] X. Nie, E. Ben-Naim, and S. Y. Chen, Europhys. Lett. **51**, 679 (2000).